Basics on Digital Signal Processing

Introduction

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Outline of the Course

1. Introduction (sampling – quantization)
2. Signals and Systems
3. Z-Transform
4. The Discreet and the Fast Fourier Transform
5. Linear Filter Design
6. Noise
7. Median Filters

Analog & digital signals

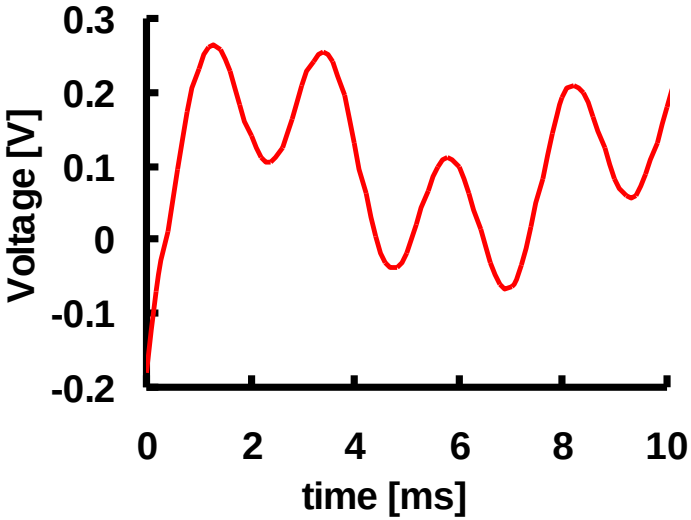
Analog

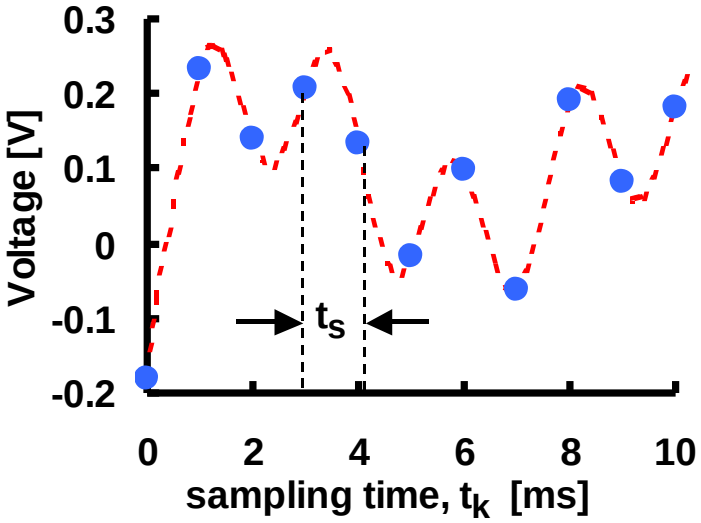
Continuous function V of *continuous* variable t (time, space etc) : V(t).

Sampled Signal

Digital

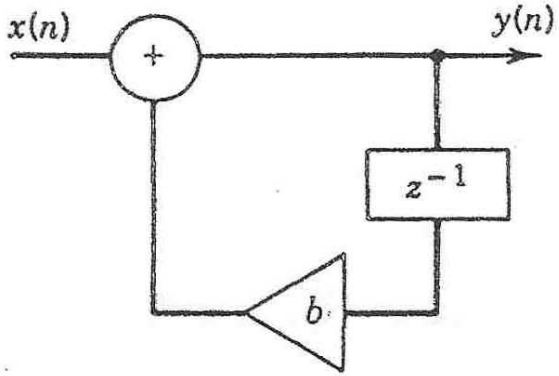
Discrete function V_k of *discrete* sampling variable t_k, with k = integer: $V_k = V(t_k)$.



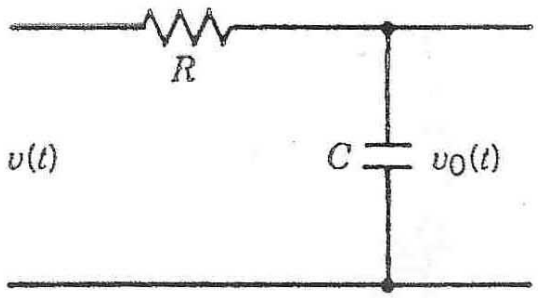


Uniform (periodic) sampling.
Sampling frequency $f_S = 1/ t_S$

Analog & digital systems



Digital lowpass filter



Analog lowpass filter

1. Discrete-time	1. Continuous-time
2. Difference equation $y(n) = by(n-1) + x(n)$	2. Differential equation $v(t) = RC\frac{dv_0(t)}{dt} + v_0(t)$
3. z-plane (Z-transform used for analysis)	3. s-plane (Laplace transform used for analysis)

Digital vs analog processing

Digital Signal Processing (DSPing)

Advantages

- More flexible.
- Often easier system upgrade.
- Data easily stored -memory.
- Better control over accuracy requirements.
- Reproducibility.
- Linear phase
- No drift with time and temperature

Limitations

- A/D & signal processors speed: wide-band signals still difficult to treat (real-time systems).
- Finite word-length effect.

DSPing: aim & tools

Applications

- Predicting a system's output.
- Implementing a certain processing task.
- Studying a certain signal.

Hardware

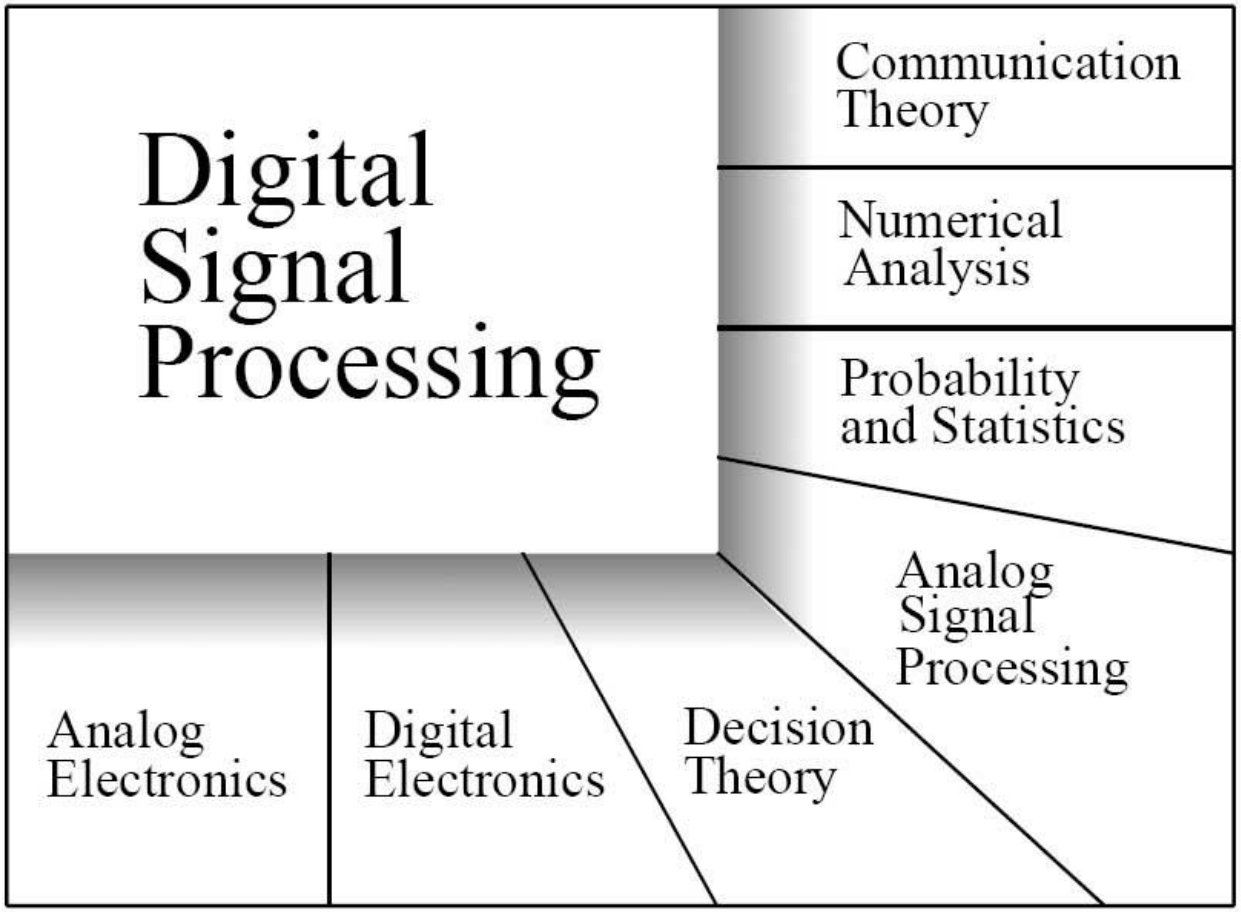
- General purpose processors (GPP), μ-controllers.
- Digital Signal Processors (DSP).
- Programmable logic (PLD, FPGA).

Fast / Faster > real-time DSPing

Software

- Programming languages: Pascal, C / C++ ...
- "High level" languages: Matlab, Mathcad, Mathematica…
- Dedicated tools (ex: filter design s/w packages).

Related areas

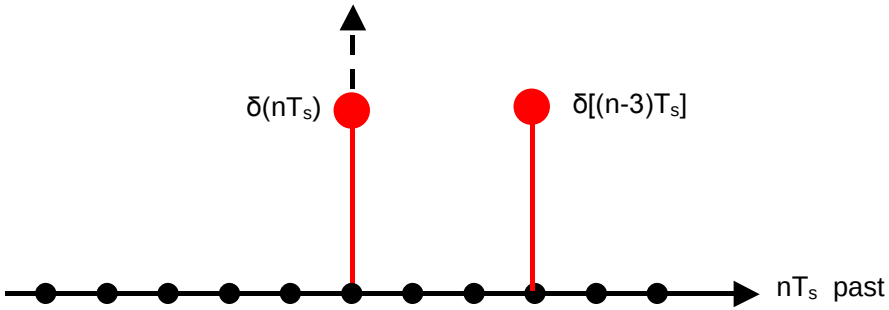


Applications

DSP

- **Space**
  - -Space photograph enhancement
  - -Data compression
  - -Intelligent sensory analysis by remote space probes
- **Medical**
  - -Diagnostic imaging (CT, MRI, ultrasound, and others)
  - -Electrocardiogram analysis
  - -Medical image storage/retrieval
- **Commercial**
  - -Image and sound compression for multimedia presentation
  - -Movie special effects
  - -Video conference calling
- **Telephone**
  - -Voice and data compression
  - -Echo reduction
  - -Signal multiplexing
  - -Filtering
- **Military**
  - -Radar
  - -Sonar
  - -Ordnance guidance
  - -Secure communication
- **Industrial**
  - -Oil and mineral prospecting
  - -Process monitoring & control
  - -Nondestructive testing
  - -CAD and design tools
- **Scientific**
  - -Earthquake recording & analysis
  - -Data acquisition
  - -Spectral analysis
  - -Simulation and modeling

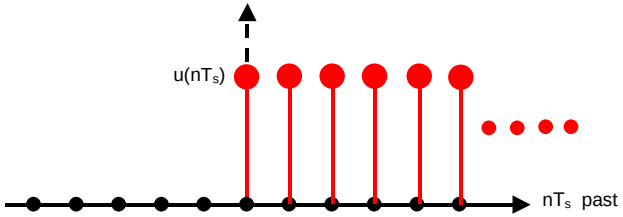
Important digital signals



Unit Impulse or Unit Sample.

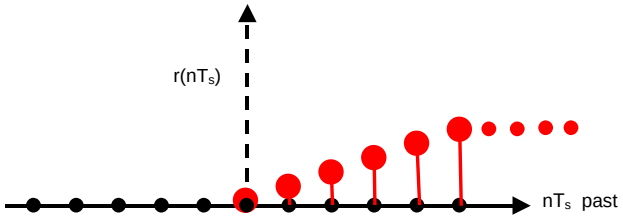
The most important signal for two reasons

δ(n)=1 for n=0

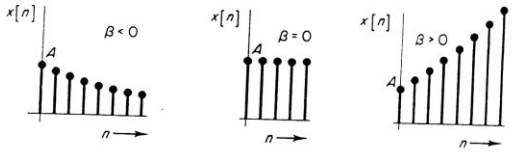


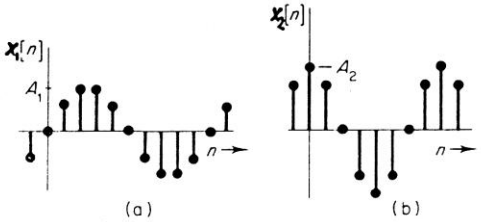
Unit Step u(n)=1 for n≥0

δ(n)=u(n)-u(n-1)



Unit Ramp r(n)=nu(n)





Digital system example

General scheme !

Sometimes steps missing

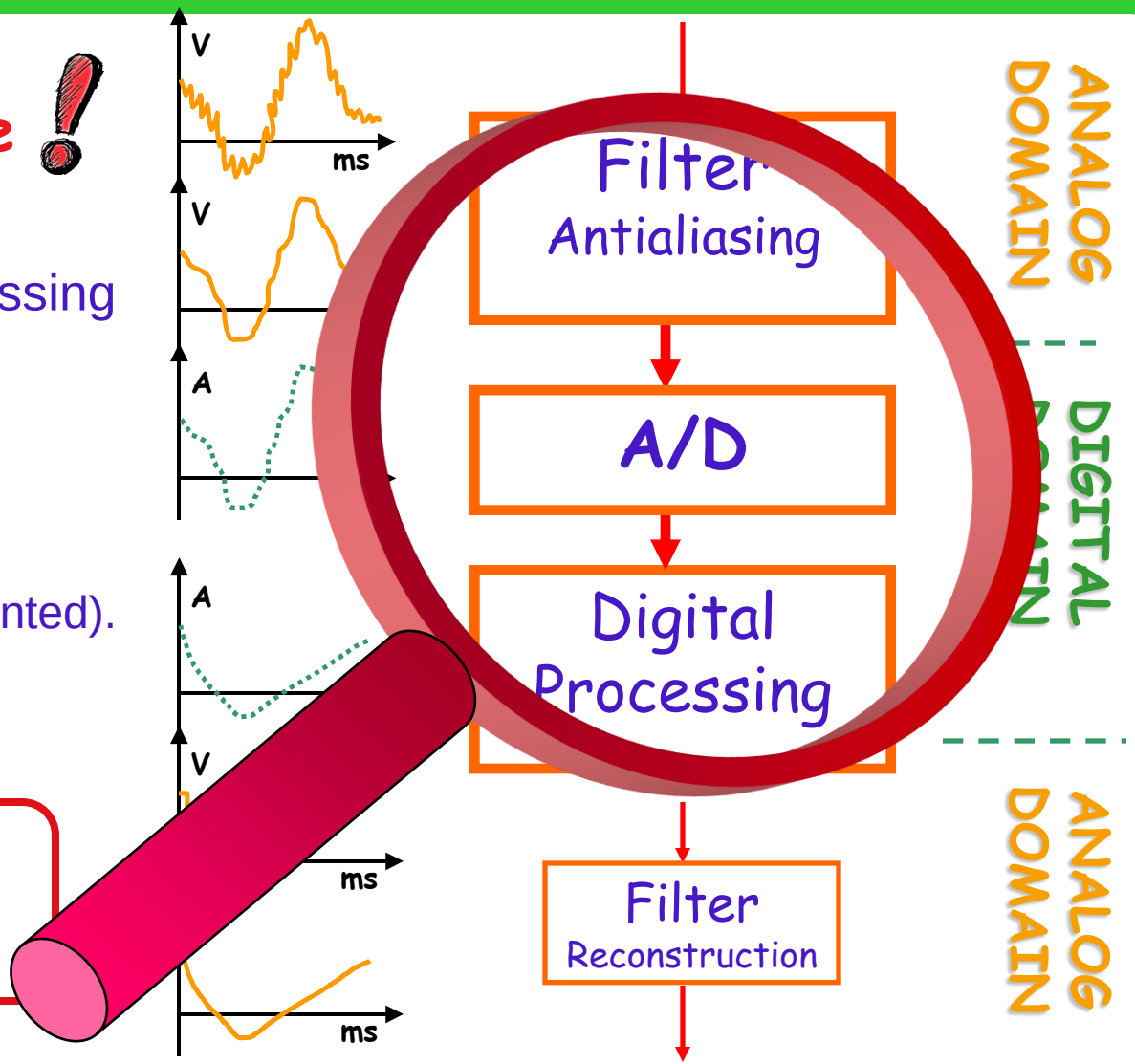
- **Filter + A/D**

  (ex: economics);

- **D/A + filter**

  (ex: digital output wanted).

Topics of this lecture.



Digital system implementation

ANALOG INPUT

↓

Antialiasing Filter

↓

A/D

↓

Digital Processing

↓

DIGITAL OUTPUT

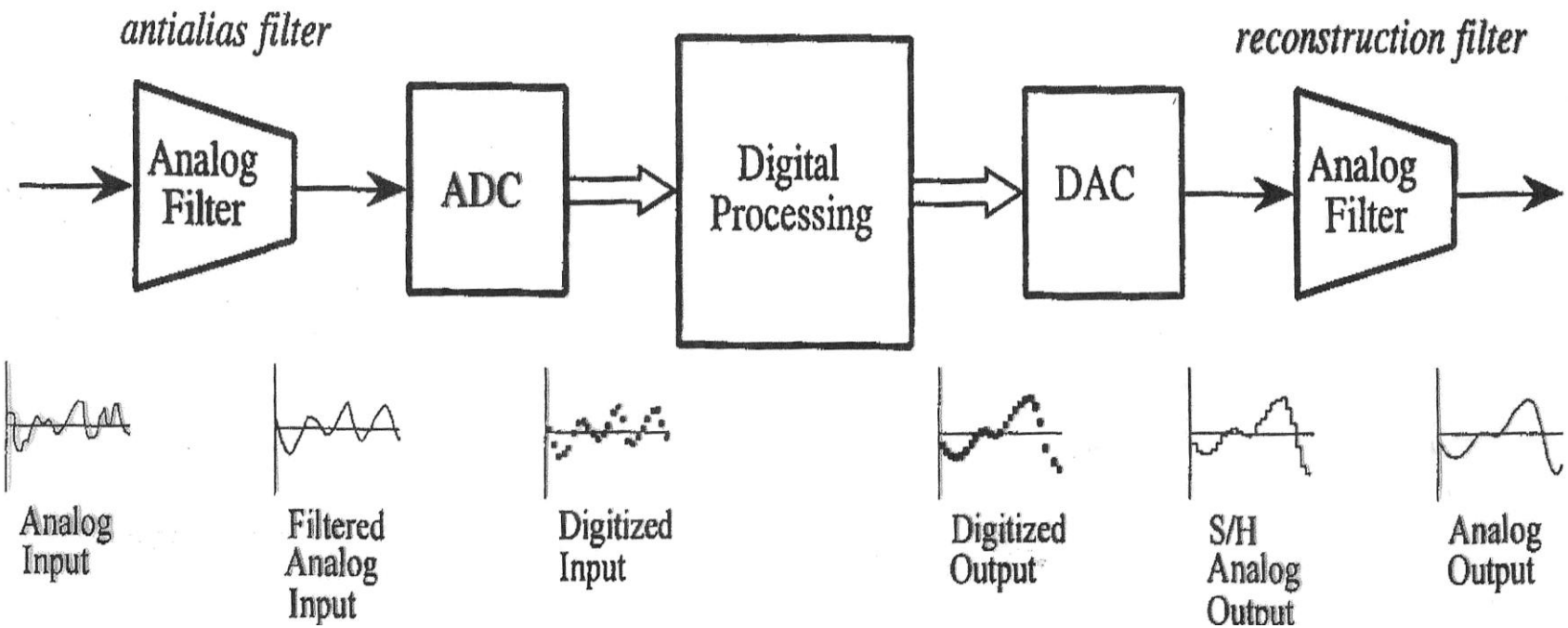
KEY DECISION POINTS:

Analysis bandwidth, Dynamic range

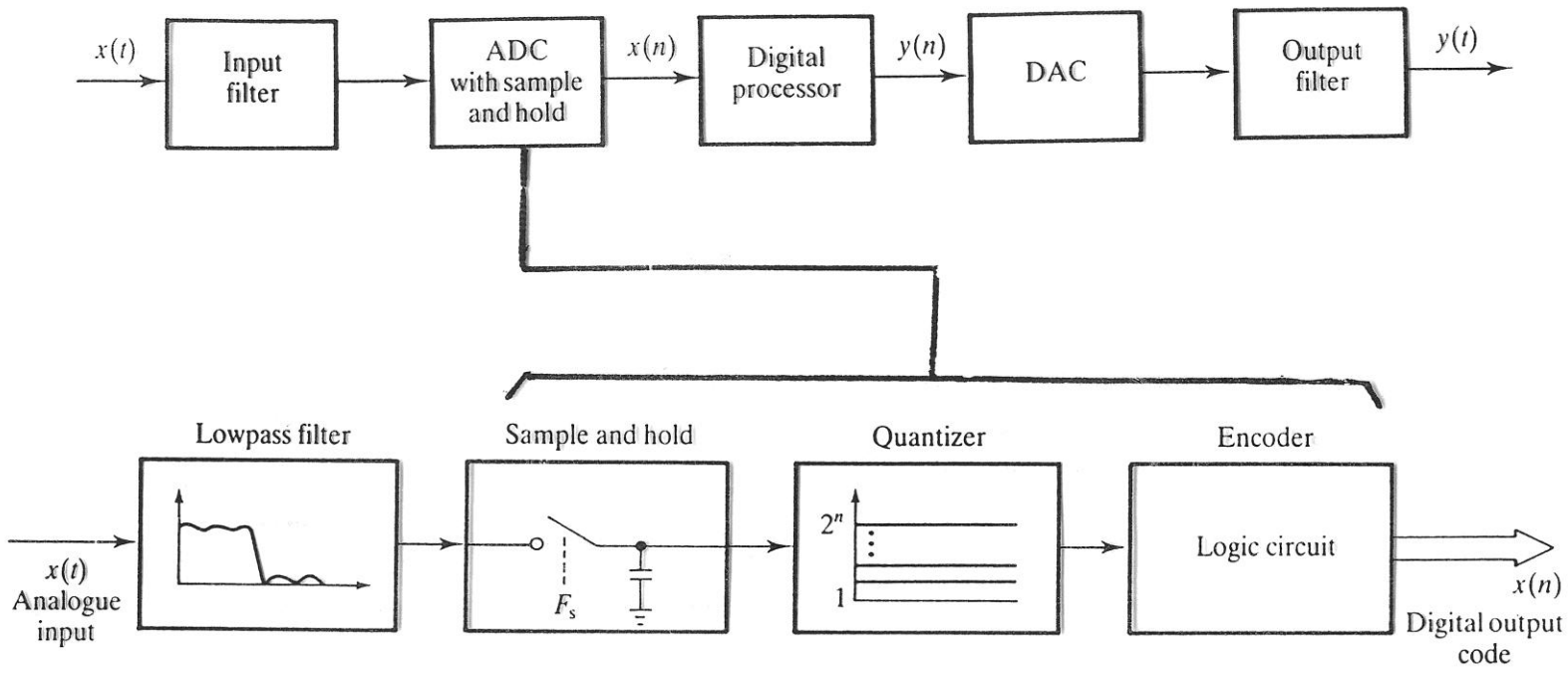
- Pass / stop bands. (1)
- Sampling rate. (1)
- No. of bits. Parameters. (2)
- Digital format. (3)

What to use for processing?

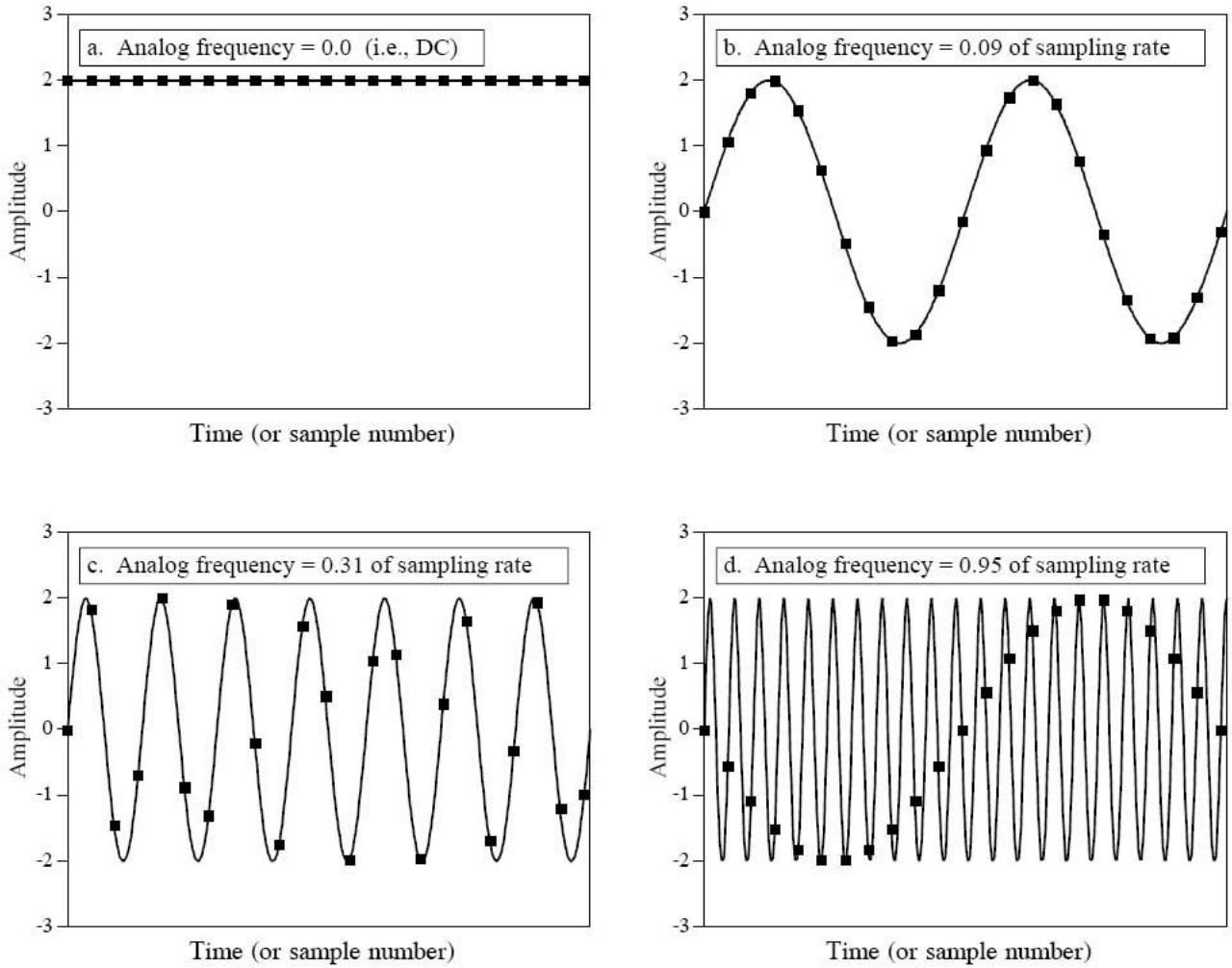
AD/DA Conversion – General Scheme



AD Conversion - Details



Sampling


a. Analog frequency = 0.0 (i.e., DC)
b. Analog frequency = 0.09 of sampling rate
c. Analog frequency = 0.31 of sampling rate
d. Analog frequency = 0.95 of sampling rate
Amplitude
Time (or sample number)
3
2
1
0
-1
-2
-3

1 Sampling

How fast must we sample a continuous signal to preserve its info content?

Ex: train wheels in a movie.

25 frames (=samples) per second.

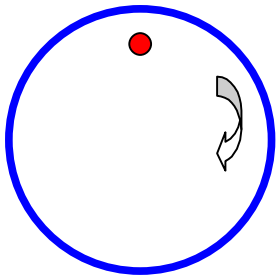
Train starts ➡ wheels 'go' clockwise.

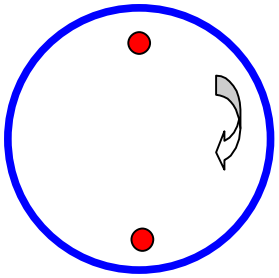
Train accelerates ➡ wheels 'go' counter-clockwise.

Why?

Frequency misidentification due to low sampling frequency.

Rotating Disk





How fast do we have to instantly stare at the disk if it rotates with frequency 0.5 Hz?

① The sampling theorem

Theo* A signal s(t) with maximum frequency f_{MAX} can be recovered if sampled at frequency $f_S > 2\, f_{MAX}$.

* Multiple proposers: Whittaker(s), Nyquist, Shannon, Kotel'nikov.

Naming gets confusing !

Nyquist frequency (rate) $f_N = 2\, f_{MAX}$ *or* f_{MAX} *or* $f_{S,MIN}$ *or* $f_{S,MIN}/2$

Example

$$s(t) = \underbrace{3\cdot\cos(50\pi t)}_{F_1} + \underbrace{10\cdot\sin(300\pi t)}_{F_2} - \underbrace{\cos(100\pi t)}_{F_3}$$

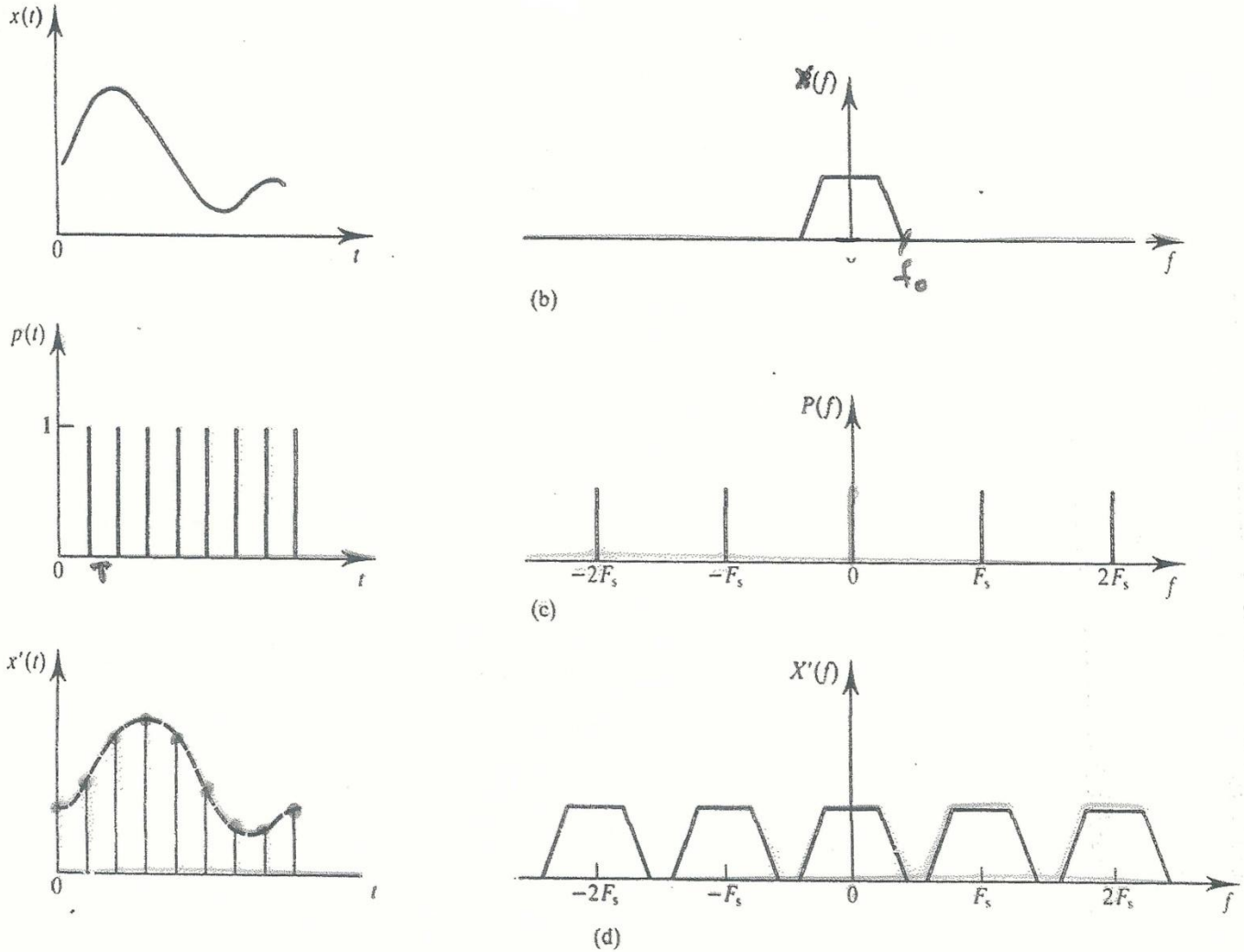
F_1=25 Hz, F_2 = 150 Hz, F_3 = 50 Hz

f_{MAX}

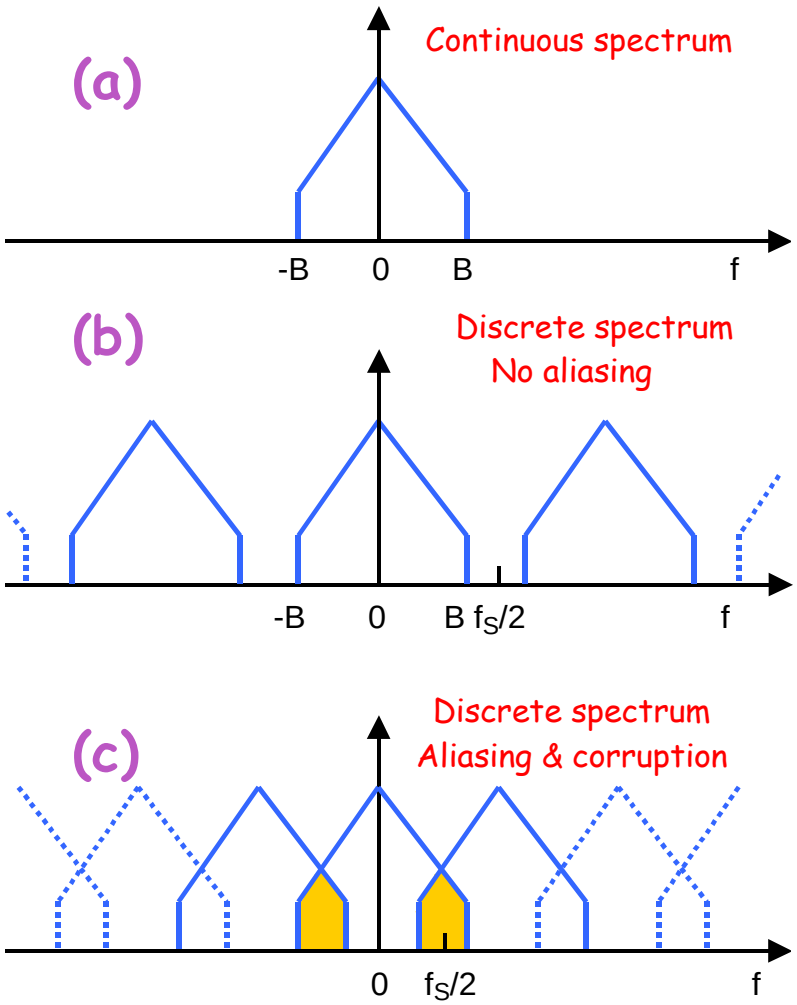
Condition on f_S?

f_S > 300 Hz

Sampling and Spectrum



① Sampling low-pass signals



(a) Band-limited signal:
frequencies in [-B, B] (f_{MAX} = B).

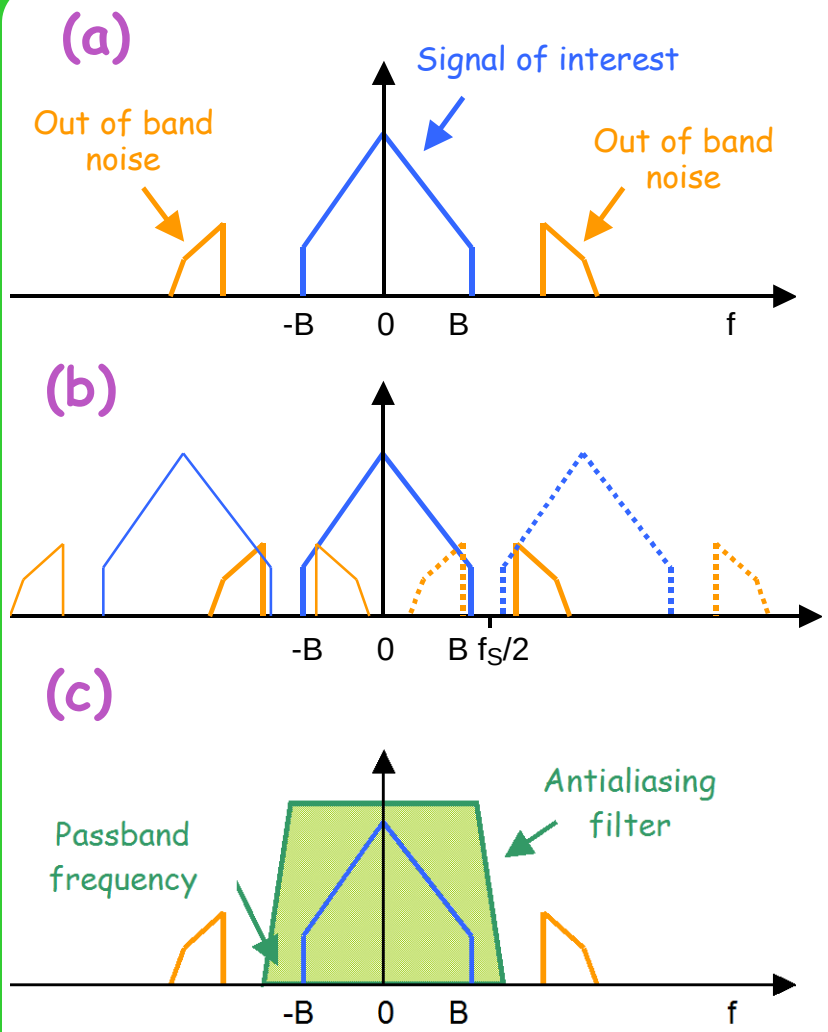
(b) Time sampling ➡ frequency repetition.

$f_S > 2\,B$ ➡ no aliasing.

(c) $f_S \leqslant 2\,B$ ➡ aliasing !

Aliasing: signal ambiguity in frequency domain

1 Antialiasing filter



(a),(b) *Out-of-band* noise can aliase into band of interest. Filter it before!

(c) Antialiasing filter

Passband: depends on bandwidth of interest.

Attenuation A_{MIN} : depends on

- ADC resolution (number of bits N).

$$A_{MIN,\,dB} \sim 6.02\,N + 1.76$$

- Out-of-band noise magnitude.

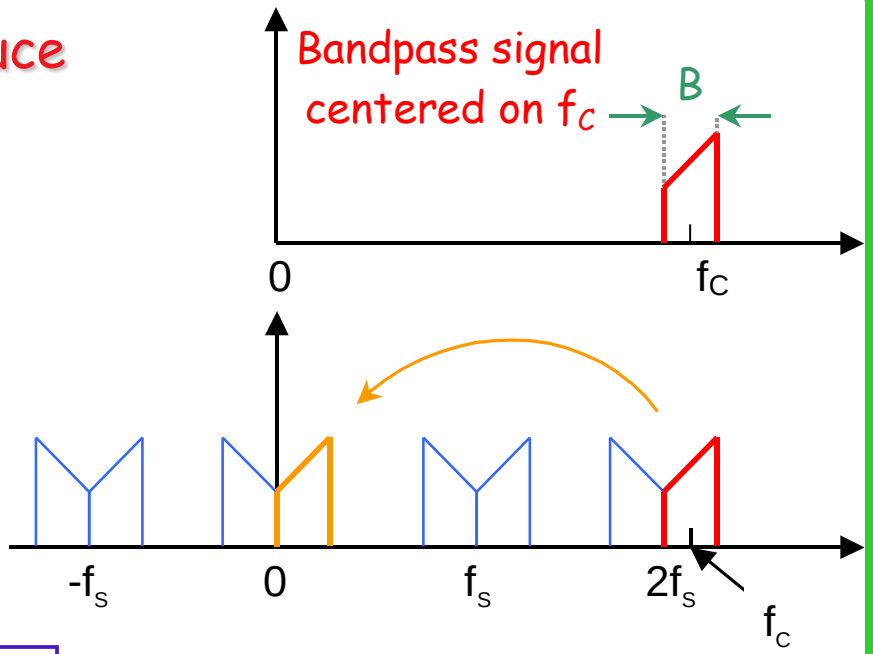
Other parameters: ripple, stopband frequency...

① Under-sampling

Using spectral replications to reduce sampling frequency f_S req'ments.

$$\frac{2\cdot f_C + B}{m+1} \le f_S \le \frac{2\cdot f_C - B}{m}$$

$m \in \mathbb{N}$, selected so that $f_S > 2B$



Example

f_C = 20 MHz, B = 5MHz

Without under-sampling f_S > 40 MHz.

With under-sampling f_S = 22.5 MHz (m=1);

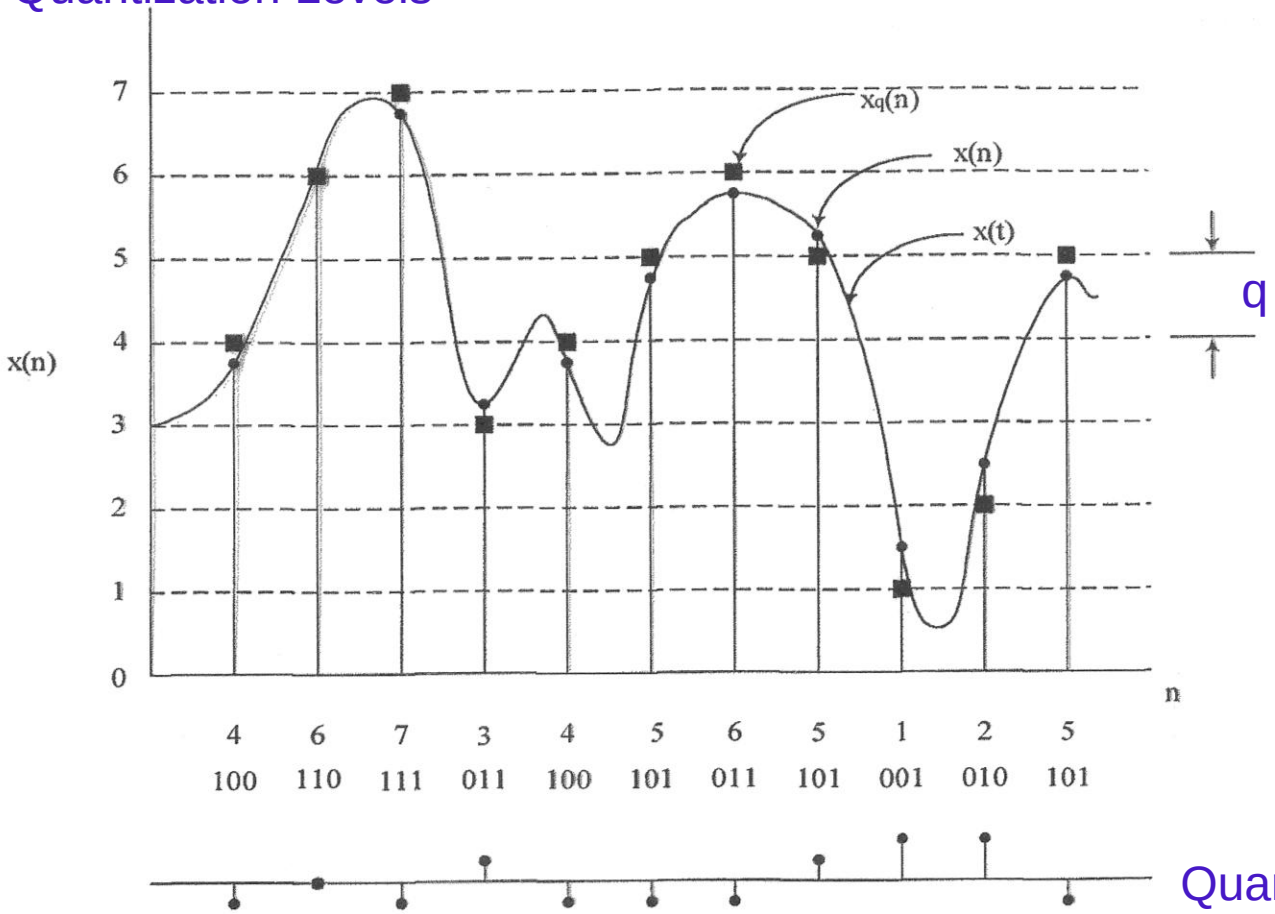
= 17.5 MHz (m=2); = 11.66 MHz (m=3).

Advantages

- Slower ADCs / electronics needed.
- Simpler antialiasing filters.

Quantization and Coding

N Quantization Levels



Quantization Noise

② SNR of ideal ADC

$$\overline{SNR}_{ideal} = 20 \cdot \log_{10}\left(\frac{RMS(input)}{RMS(e_q)}\right) \quad (1)$$

Also called SQNR
(signal-to-quantisation-noise ratio)

Assumptions

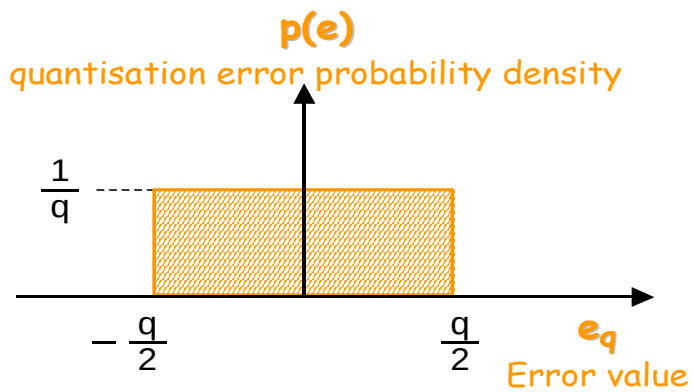
- Ideal ADC: only quantisation error e_q (**p(e)** constant, no stuck bits…)
- e_q uncorrelated with signal.
- ADC performance constant in time.

$$RMS(input) = \sqrt{\frac{1}{T} \cdot \int_0^T \left(\frac{V_{FSR}}{2} \cdot \sin(\omega t)\right)^2 dt} = \frac{V_{FSR}}{2\sqrt{2}}$$

Input(t) = ½ V_{FSR} sin(ω t).

$$RMS(e_q) = \sqrt{\int_{-q/2}^{q/2} e_q^2 \cdot p(e_q) de_q} = \frac{q}{\sqrt{12}} = \frac{V_{FSR}}{2^N \cdot \sqrt{12}}$$

(sampling frequency $f_S = 2\ f_{MAX}$)



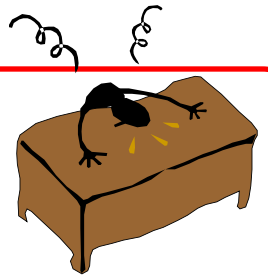
② SNR of ideal ADC - 2

Substituting in (1) :

$$\overline{SNR}_{ideal} = 6.02 \cdot N + 1.76 \, [dB] \quad (2)$$

One additional bit ➡ SNR increased by 6 dB

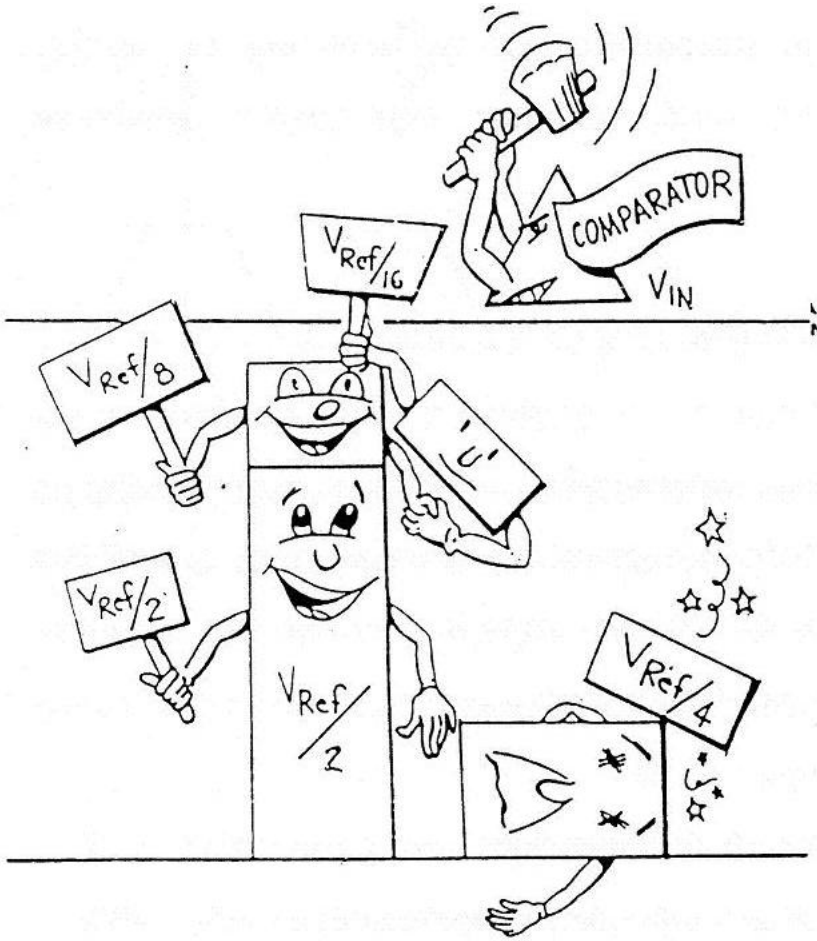
Real SNR lower because:

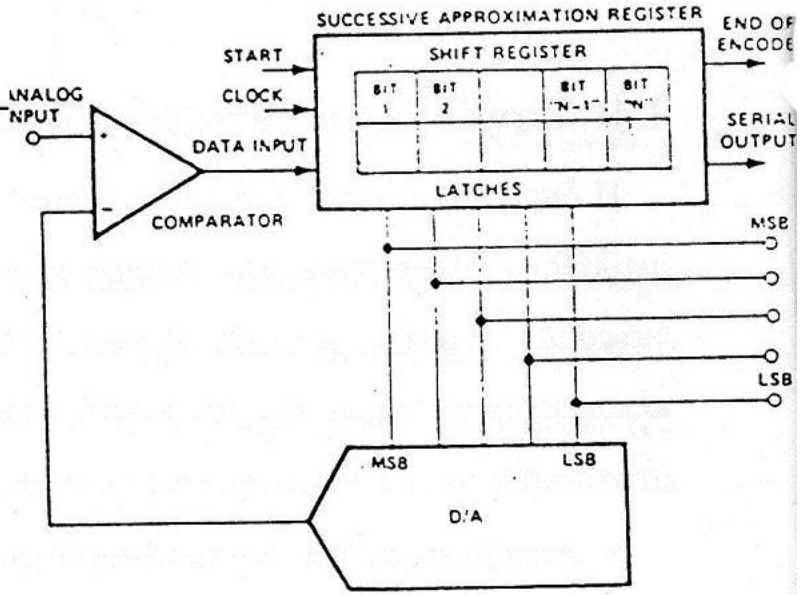


- Real signals have noise.
- Forcing input to full scale unwise.
- Real ADCs have additional noise (aperture jitter, non-linearities etc).

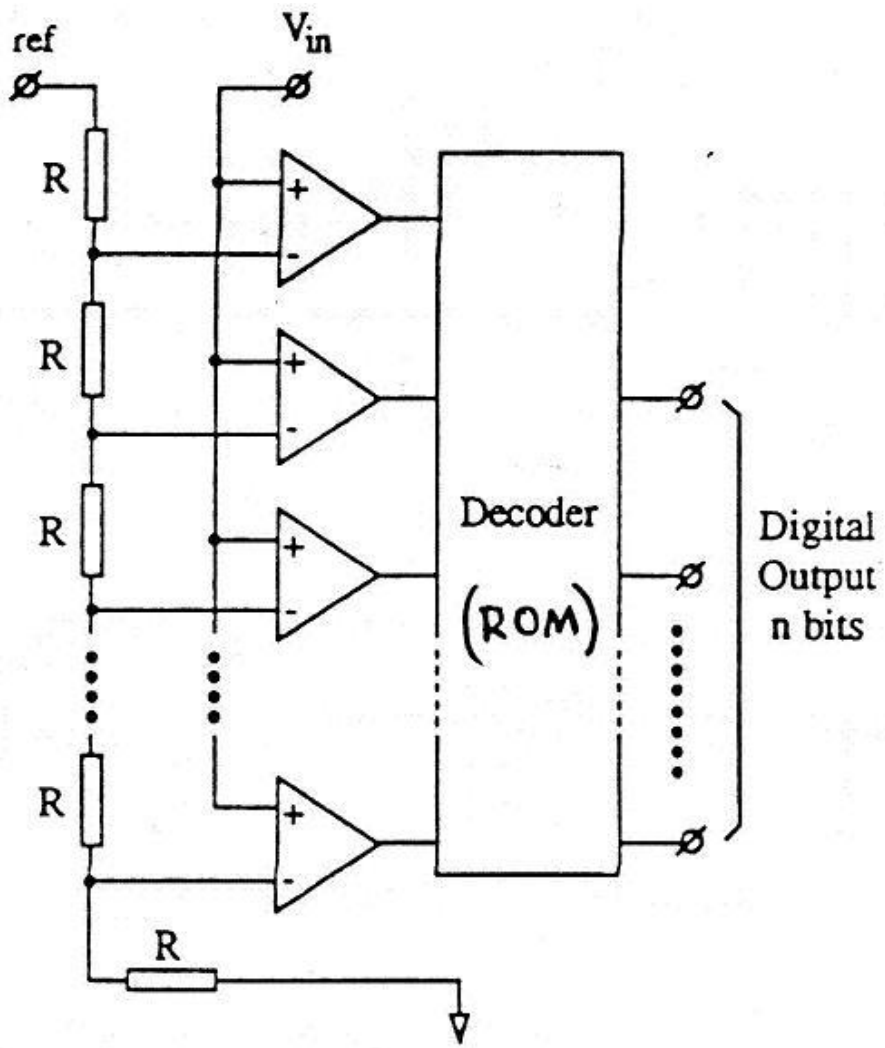
Actually (2) needs correction factor depending on ratio between sampling freq & Nyquist freq. Processing gain due to oversampling.

Coding - Conventional

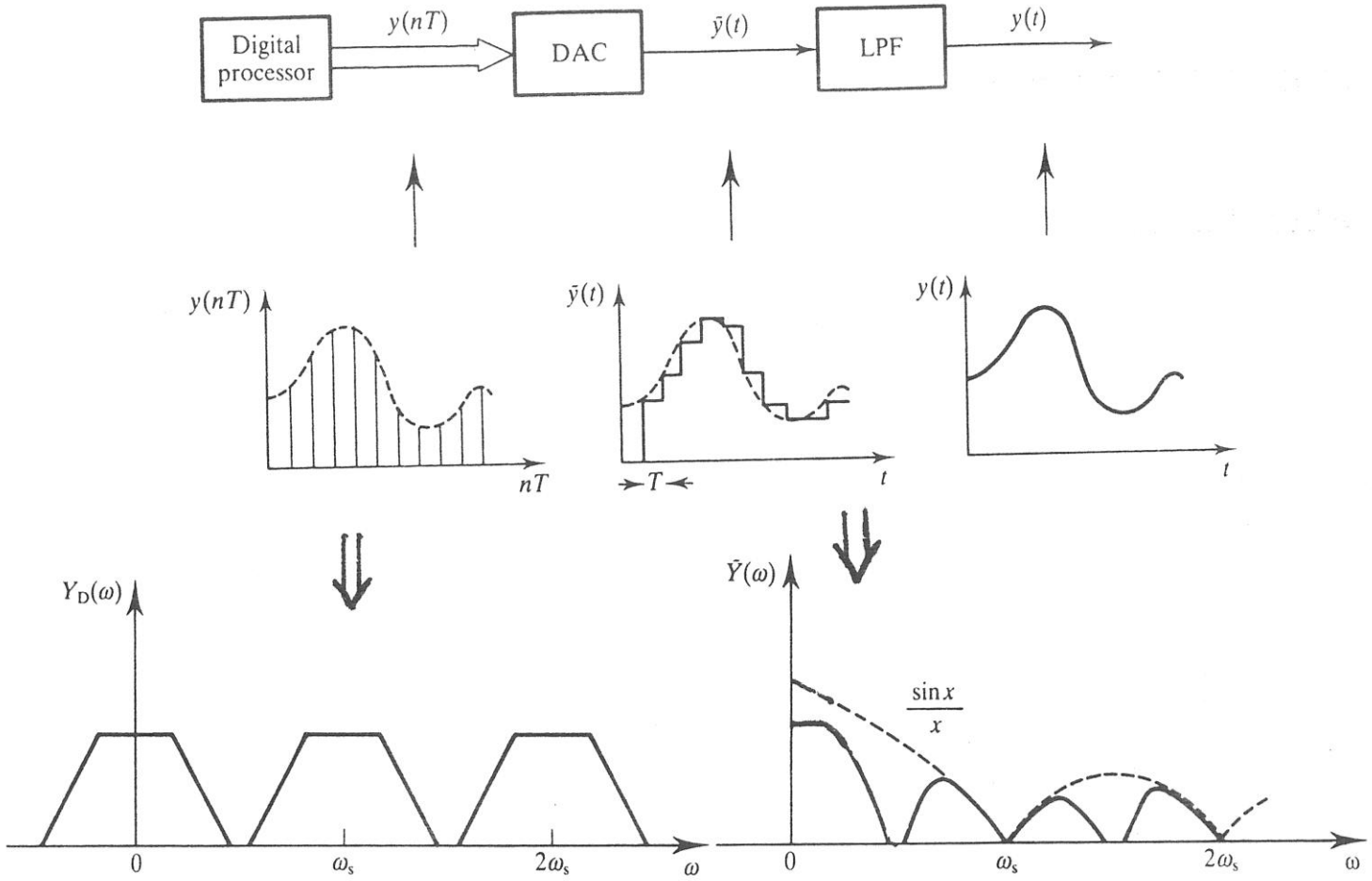




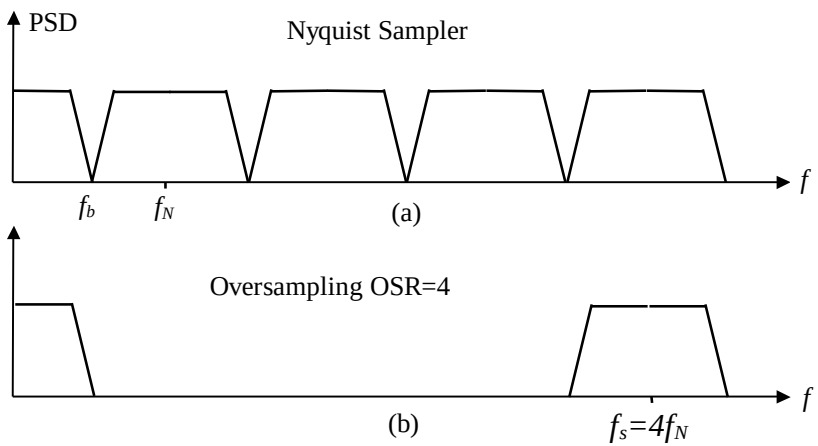
Coding – Flash AD



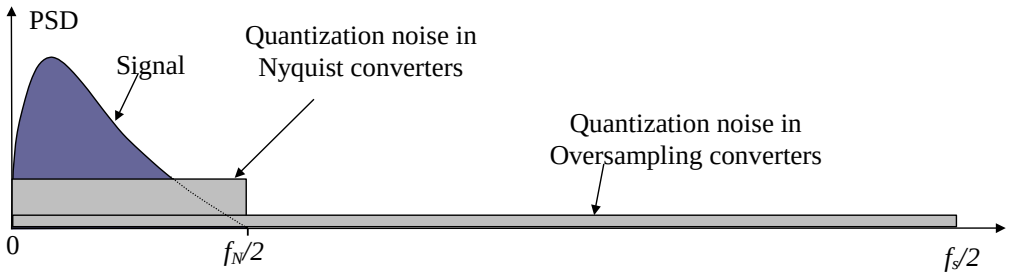
DAC process


Digital processor
y(nT)
DAC
ȳ(t)
LPF
y(t)
y(nT)
nT
ȳ(t)
T
t
y(t)
t
Y_D(ω)
0
ω_s
2ω_s
ω
Ȳ(ω)
sin x / x
0
ω_s
2ω_s
ω

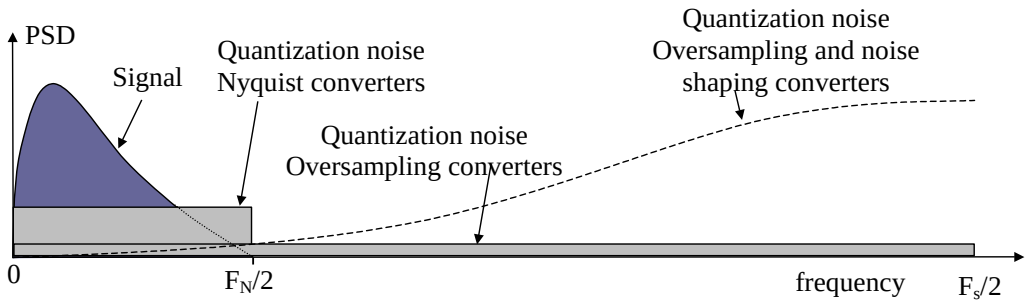
Oversampling – Noise shaping



The oversampling process takes apart the images of the signal band.



When the sampling rate increases (4 times) the quantization noise spreads over a larger region. The quantization noise power in the signal band is 4 times smaller.



Spectrum at the output of a noise shaping quantizer loop compared to those obtained from Nyquist and Oversampling converters.

Digital Systems

A discreet-time system is a device or algorithm that operates on an input sequence according to some computational procedure

It may be

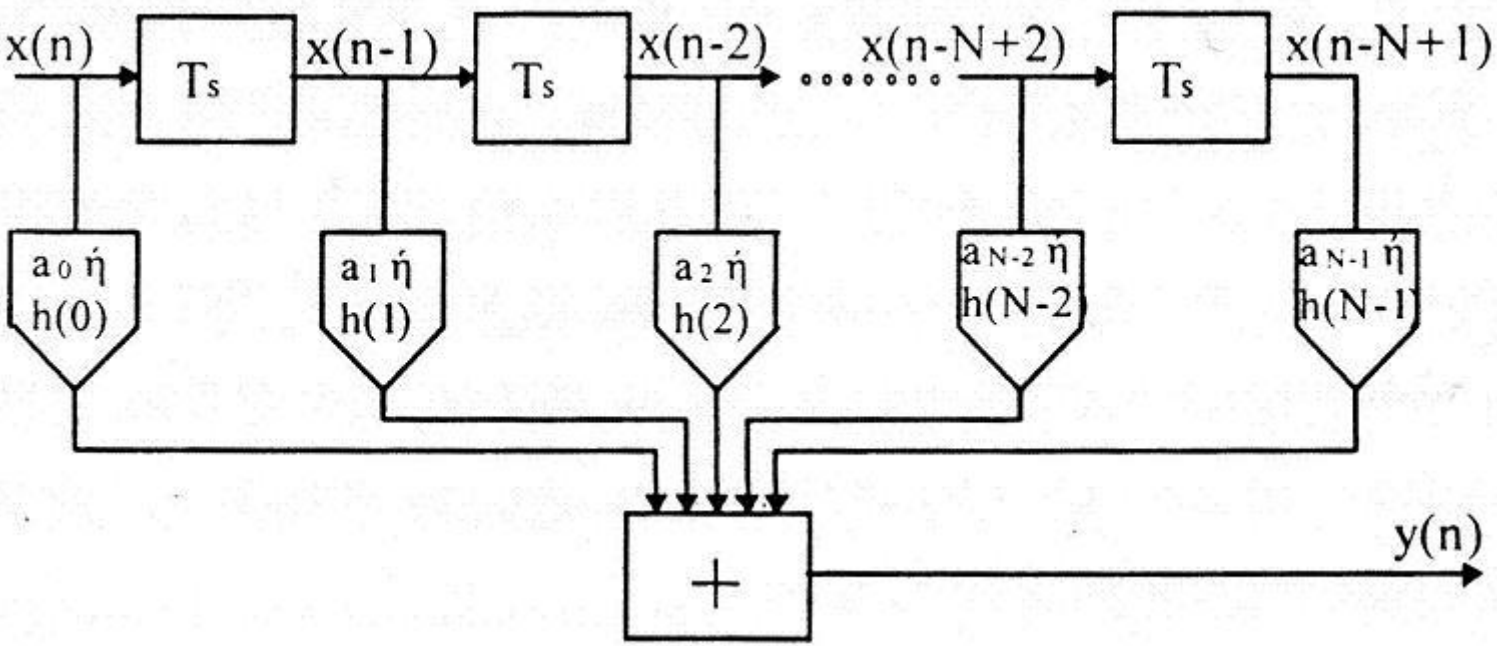
- A general purpose computer
- A microprocessor
- dedicated hardware
- A combination of all these

Linear, Time Invariant Systems

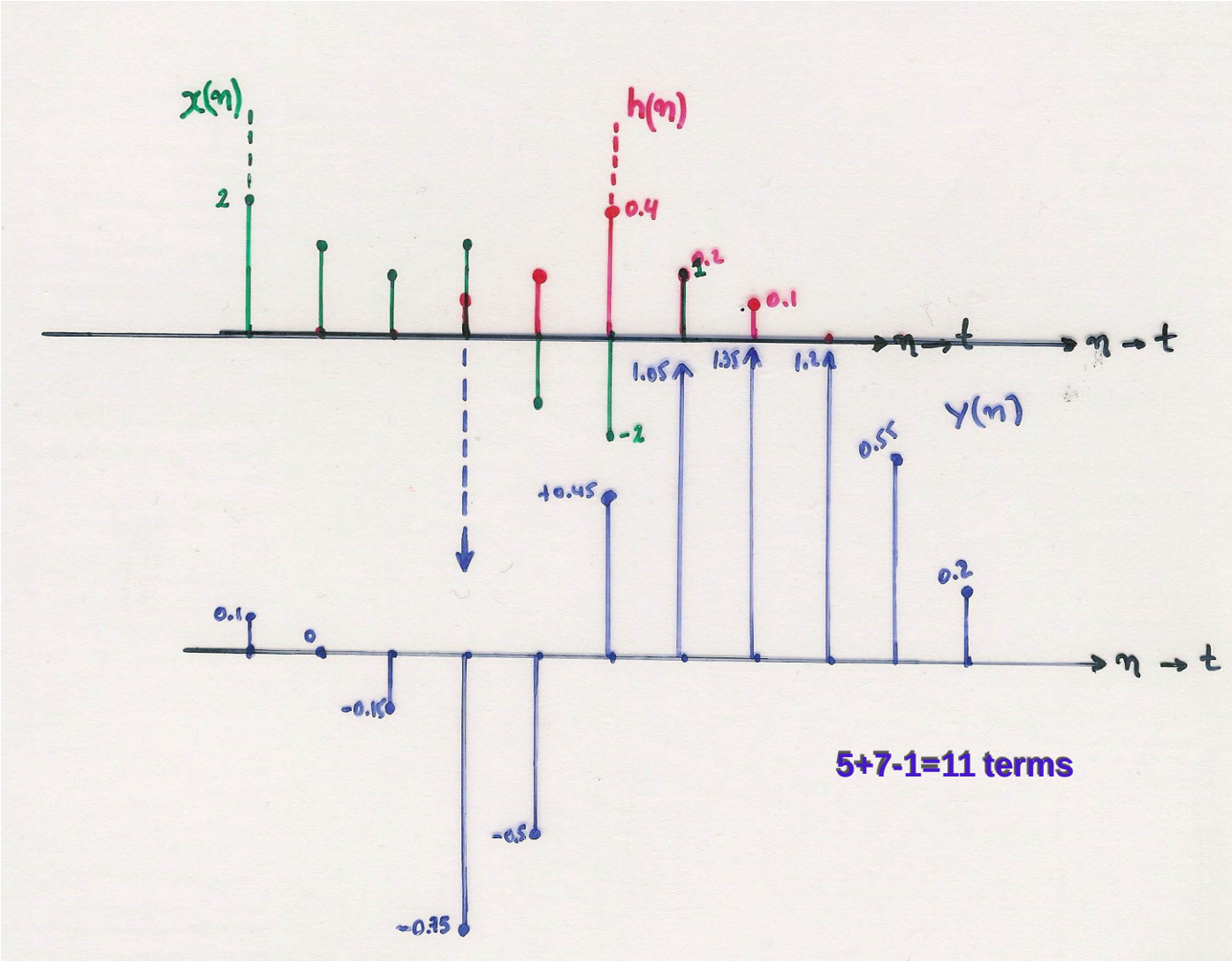
System Properties
- linear
- Time Invariant
- Stable
- Causal

$$y(n) = \sum_{k=0}^{N} a_k x(n-k)$$

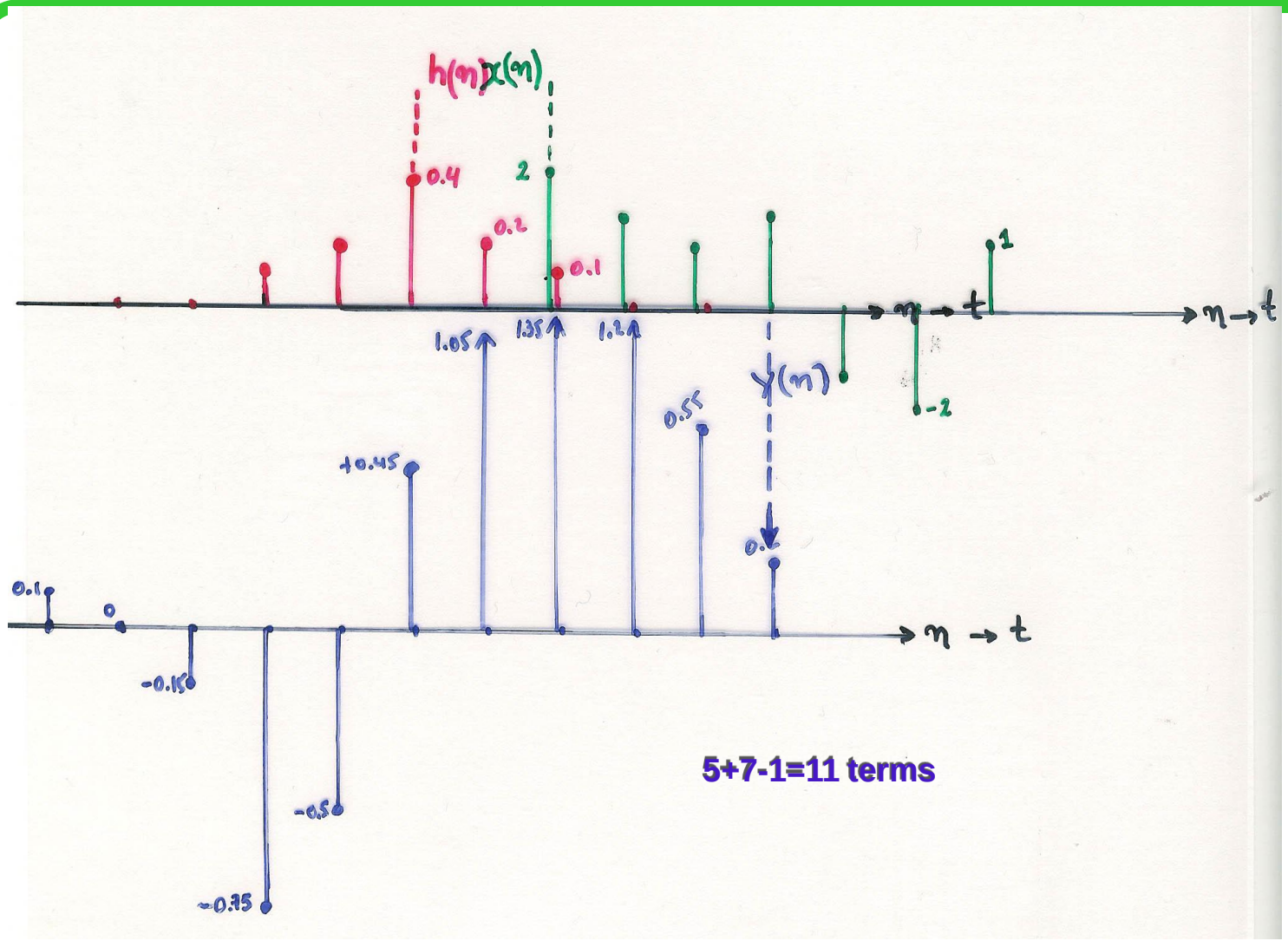
Convolution



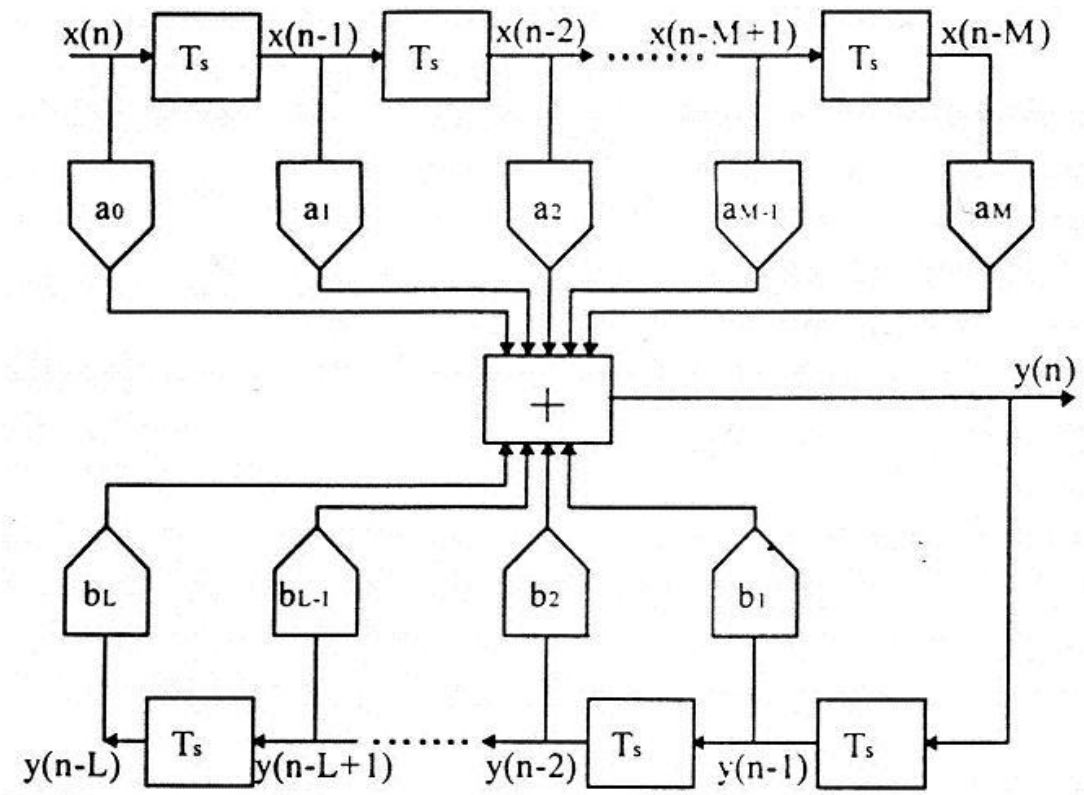
Linear Systems - Convolution



Linear Systems - Convolution

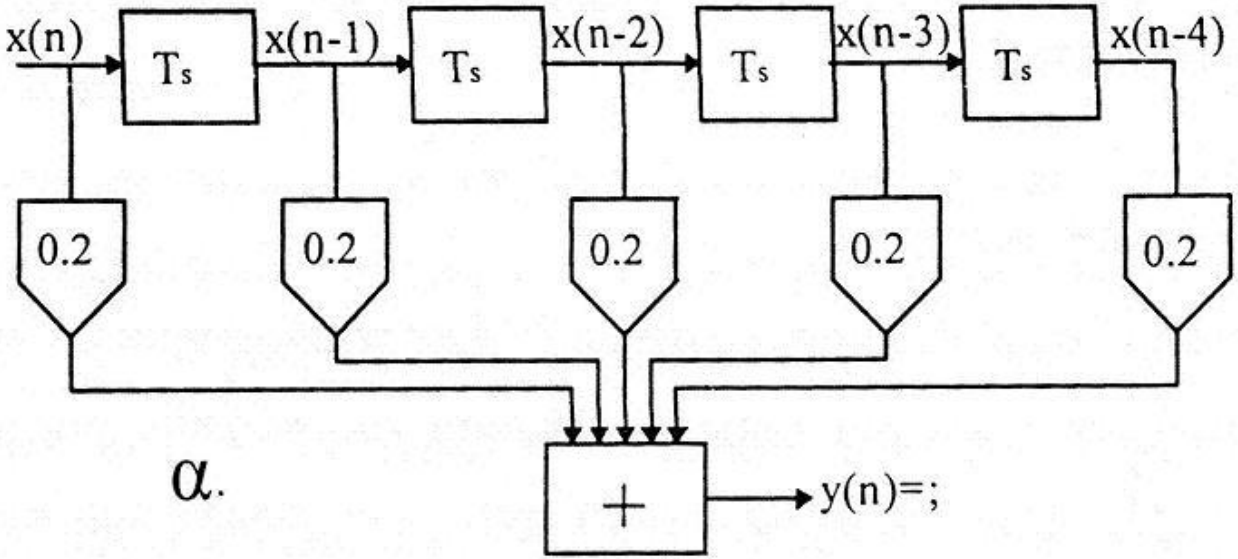


General Linear Structure

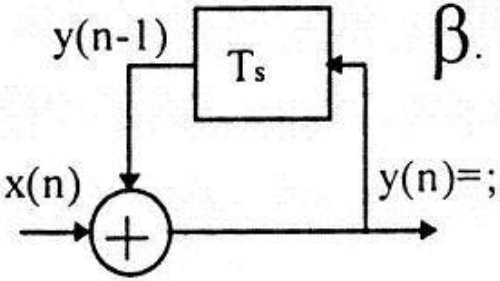


$$y(n) = \sum_{k=0}^{M} a_k x(n-k) - \sum_{k=1}^{L} b_k y(n-k)$$

Simple Examples

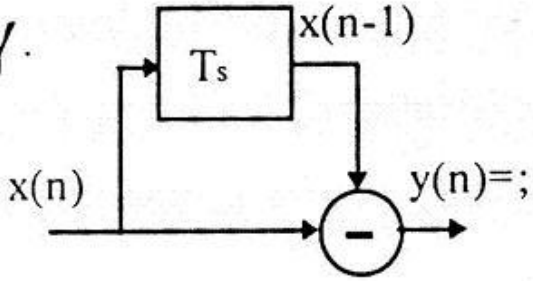


α.



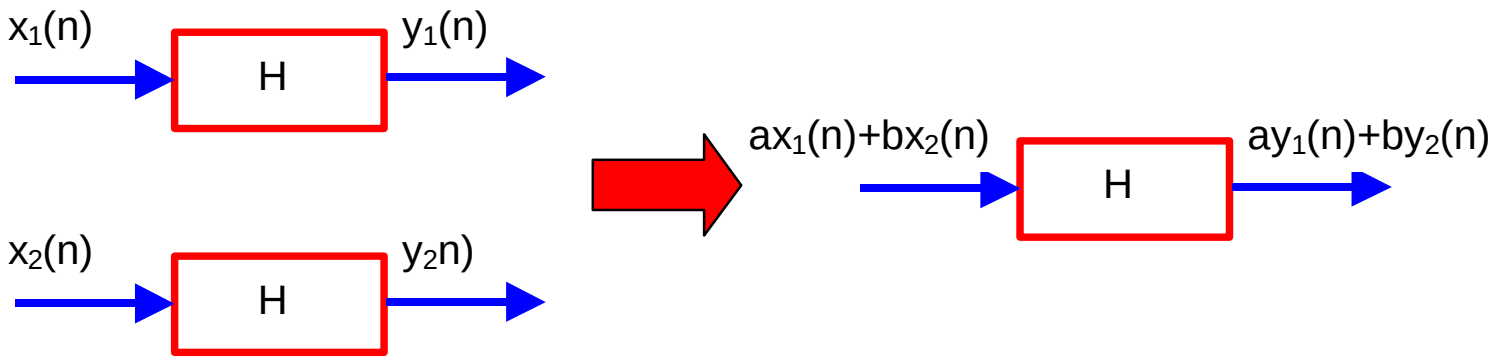
β.

γ.

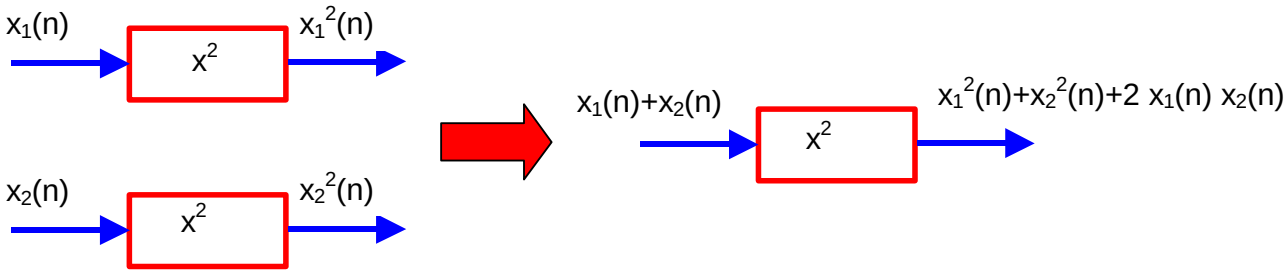


Linearity – Superposition – Frequency Preservation

Principle of Superposition



Principle of Superposition ⇒ Frequency Preservation



Non-linear

If $y(n)=x^2(n)$ then for $x(n)=\sin(n\omega)$ $y(n)=\sin^2(n\omega)=0.5+0.5\cos(2n\omega)$

The END

Have a nice Weekend

Back on Tuesday